Symmetry playing with mirrors

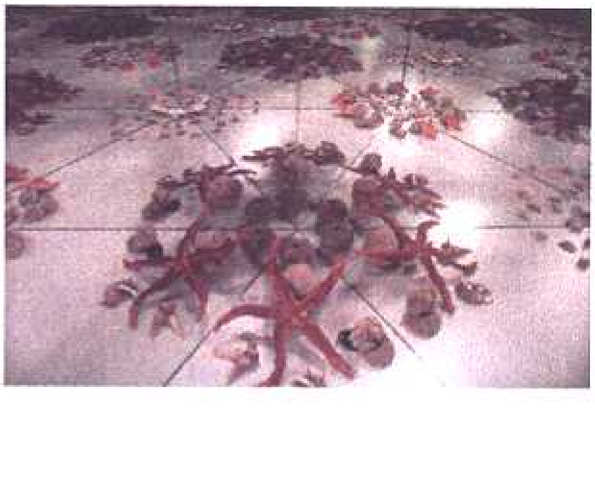
This brochure introduces the objects in the exhibition and the leaflets and worksheets that accompany it.

The underlying idea behind the exhibition is encapsulated in the five posters entitled "Symmetry". The first three posters show images, taken from a variety of sources, that all evoke symmetry: from the Arabic mosaics of the Alhambra to the landscape of the Rocky Mountains reflected in a lake, from lace to flowers. Some of these images are duplicated on the other two posters: these are the ones that can be reproduced in a mirror chamber, and they are separated according to the shape of the mirror chamber in which they can be made.

The three triangular mirror chambers on the tables are the principal tools of the exhibition. Each chamber is made of three mirrors perpendicular to the plane of the table on which it rests, and it isolates a triangle in the plane. The bases of the three chambers are: an equilateral triangle, an isosceles right-angled triangle, and a right-angled triangle with angles of 30 and 60 degrees. The main aim of the exhibition is to get visitors to understand what the patterns obtained from each mirror chamber have in common, and also what makes the patterns obtained from the various mirror chambers different.

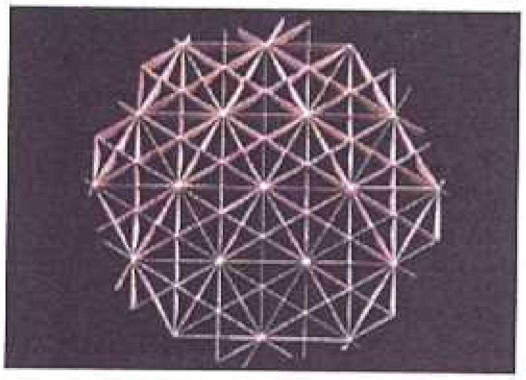
Some questions are posed to help visitors develop their intuition. On the sides of each chamber there are illustrations of tessellations that can be constructed in that chamber (they must select the correct tiles to place in it). Other worksheets show more illustrations that visitors can try to reproduce - now they must also decide which chamber should be used. So, gradually, the exhibition leads the visitors to explore which type of symmetry governs a given pattern.

It is not necessary for visitors to perform the suggested experiments in order to understand the type of symmetry common to the different patterns in each mirror chamber: the arrangement of the reflected images remains constant, whatever object is placed between the



mirrors, be it a scarf, a bunch of flowers or a pair of ping-pong balls.
The grids above the triangular chambers clarify what happens: one triangle (which represents the mirror chamber) is highlighted, and the grid itself (indefinitely extended) represents what it is seen in the chamber when it is empty.

To understand the geometry underlying



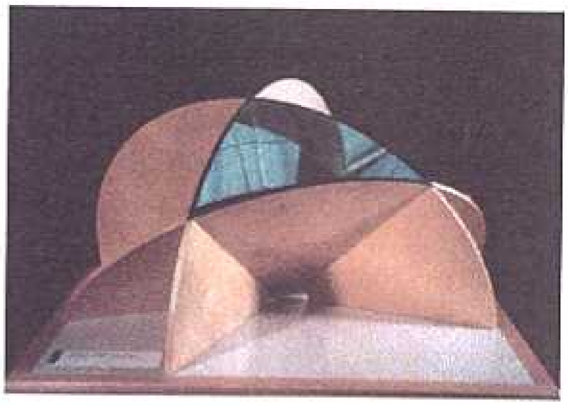
the infinite patterns taking shape in the mirror chambers, it is useful to start analysing what happens with only one mirror, or with two of them.
One table of the exhibition asks visitors to use a mirror to search for mirror lines in a drawing; others present two parallel or two incident mirrors to guide visitors towards the rules for the composition of two reflections. The explanatory sheets [1], [2] and [3] cover these topics.

There are also two smaller mirror chambers, with square bases. Visitors can draw whatever they want and observe directly what happens when their drawing is placed into the chamber.

The three posters on mosaics complete the journey for those who want to know more: they show 17 different symmetry types for a mosaic, that is for a drawing that repeats itself periodically in the plane in more than one direction. (The tilings that can be seen in the mirror chambers provide examples of some of these 17 types.) The fact that these 17 types are the only ones possible is a deep and even non-trivial mathematical result, and it is surprising that artists from all over the world managed to produce examples for each of these cases long before mathematicians tackled the problem.
Explanatory sheets [4] and [5] explore the topic of plane tilings, while sheet [6] is about the 17 symmetry types for mosaics.

There is a situation analogous to that described for mosaics for the patterns repeated periodically in one direction only, the so-called frieze patterns. In this case, there are only seven possibilities, and examples are shown on a poster. All these possibilities can be obtained by cutting a strip of paper that has been suitably folded or rolled up. A display case shows how this can be done.
Sheets [7] and [9] cover this topic, while sheet [8] shows some examples of the seven friezes in African arts and crafts.

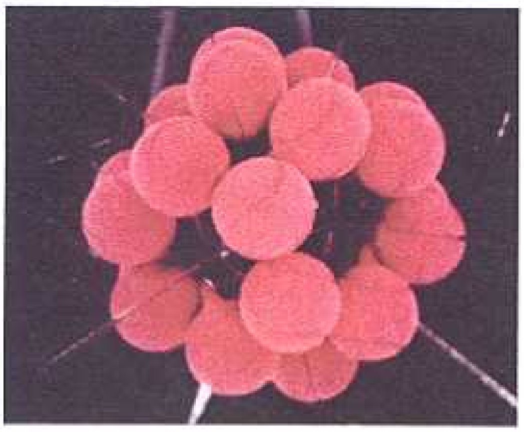
It is natural to ask whether mirrors can also be used to classify the symmetry type of solid objects. The answer is suggested by the three 3-dimensional kaleidoscopes that are another focal point of the exhibition.
They work in the same way as the triangular mirror chambers: if an object (for example a little ball) is placed into the kaleidoscope, its reflected images will be arranged about a centre and create something 3-dimensional.



A ball is suspended above each kaleidoscope. Each ball is subdivided into (spherical) triangles, all of which are equivalent to each other. It represents for the kaleidoscopes what the grid is for the plane mirror chambers. If we imagine lining one of these triangles with mirrors (each mirror represents a plane through the centre of the ball and one side of the triangle), we obtain the corresponding

kaleidoscope.
Sheet [10] explores the link between the 2-dimensional and 3-dimensional situations.

By placing different objects in the same kaleidoscope, different virtual objects are obtained, but they all share the same symmetry type, while in different kaleidoscopes images with different symmetry are seen. Such observations can be made by using any object, but a ball is particularly instructive for showing how many copies are made and how they are distributed. By the side of each kaleidoscope there are also some "bricks" that enable visitors to produce some



virtual polyhedra. These include the regular polyhedra, which are also shown in the three posters corresponding to the kaleidoscopes.

For example, one problem is to discover how to generate a cube: which brick should be placed in which kaleidoscope to reconstruct the image of a whole cube? Another table of the exhibition aims to illustrate how the shape of a brick can be worked out. The table contains three pairs of incident mirrors at different angles, and various polyhedra representing fragments of a cube. By placing the right fragment in the right way against one mirror (if it is half a cube) or against two mirrors (in the other cases), the whole cube can be seen. More information can be found in sheets [11] and [12].

There are also some small 3-dimensional kaleidoscopes, but objects must not be placed in them: they are already cut at the bottom to reveal a polyhedron. By looking carefully, visitors may notice that kaleidoscopes of the same colour are related: the angles between the various mirrors are the same; only the way they are cut at the bottom changes. The polyhedra that they show can all be reconstructed in the large kaleidoscope of the same colour (once the right brick is found for each polyhedron). For those interested in the topic of polyhedra, there is more information in sheets [13] to [19].
The sheet [20] is about symmetry in nature.

Sheets of the exhibition

[1] ***Reflections*** by P. Cereda

[2] ***Rosettes*** by P. Cereda

[3] ***Alphabet at the Mirror*** by P. Testi Saltini, in preparation

[4] ***Tessellations*** by P. Bellingeri, in preparation

[5] ***The Rules of Imagination*** by P. Bellingeri, in preparation

[6] ***Mosaics*** in preparation

[7] ***Friezes*** by P. Cereda, in preparation

[8] ***Friezes in African Arts and Crafts*** by P. Testi Saltini, in preparation

[9] ***The Rhythm of Colours*** by P. Cereda, in preparation

[10] ***Plane and Spherical Triangles***

[11] ***Symmetries of the Cube*** by C. Vezzani

[12] ***The Family of Symmetries of the Cube*** by C. Vezzani

[13] ***The Five Regular Polyhedra*** by C. Vezzani

[14] ***More on the Five Regular Polyhedra*** by C. Vezzani

[15] ***Why Only Five?*** by C. Vezzani

[16] ***Stellated Polyhedra*** by E. Frigerio

[17] ***Uniform Polyhedra*** in preparation

[18] ***More on Uniform Polyhedra*** in preparation

[19] ***Euler's Formula*** by A. Cazzola, in preparation

[20] ***Symmetry in Nature*** by C. Vezzani

[21] ***An "Instrument of Wonders"*** by André Gide

(translated by Elisabetta Beltrami and Peter Cromwell)